

Dynamic $0 - \pi$ transition induced by pumping mechanism

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Abstract

Using Nambu \otimes spin space Keldysh Green's function approach, we present a nonequilibrium charge and spin pumping theory of a quantum dot in the micro-cavity coupled to two superconducting leads. It is found that the charge currents include two parts: The dissipationless supercurrent standing for the transfer of coherent Cooper pairs and the pumped quasi-particle current. The supercurrent exhibits a dynamic $0 - \pi$ transition induced by the frequency and strength of the σ_- polarized laser field. This dynamic transition is not affected by the strong Coulomb interaction. Especially, the spin current appears and is an even function of the phase difference between two superconductors when the frequency of the polarized laser field is larger than two times superconducting energy gap. Our theory serves as an extension to non-superconducting spintronics.

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The interplay between superconducting order and ferromagnetic order has attracted considerable attention recently in the proximity effects near the interface of ferromagnet/superconductor heterostructures, since it subsumes many fascinating physical phenomena at the interface and leads to potential applications that may complement non-superconducting spintronic devices (see Reviews[1, 2, 3] and Refs. therein). A interesting physical phenomenon due to the interplay of magnetic and superconducting orders is the so-called $0 - \pi$ transition in superconductor-ferromagnet-superconductor (S/F/S) junctions[4, 5, 6, 7, 8, 9] and the magnetic quantum dots[10, 11]. The existence of the π junction in layered S/F/S systems was first predicted by Bulaevskii *et al.*[4] and Buzdin *et al.*[5] to occur for certain thicknesses and exchange field energies of the F layer and was later confirmed in the experiment[6]. These investigations are not only of academic interest, but also of importance for the solid-state implementation of a quantum bit based on a superconducting loop with 0 and π Josephson junctions[12].

The present work is also related to the parametric quantum pumping[13], which refers to the transfer of electrons coherently between two reservoirs at zero bias. In particular, when the charge transfer is quantized per cycle, the pumping could be of importance in establishing a standard of electric current[14]. Therefore, charge pumping has attracted much attention since the first proposal by Thouless[15]. The first experimental demonstration of the pumping of charges was reported by Switkes *et al.*[16]. They applied sinusoidal gate voltages to an open quantum dot to pump the charge current. More recently, this pumping mechanism has been extended to different transport contexts, including the generation of spin current[17, 18, 19, 20, 21] and entangled pairs[22, 23].

In this Letter, we present a nonequilibrium charge and spin transport theory of a quantum dot under the optical micro-cavity. We find that the laser field acting as the pumping forces can induce the dynamic $0 - \pi$ transition which is not affected by the strong Coulomb interaction. Our theory goes beyond the adiabatic limit, and is valid for the arbitrary pumping frequency and temperature. More interestingly, we find the spin current as an even function of the phase difference between two superconductors is pumped when the laser frequency is larger than 2Δ .

The system we consider is a quantum dot embedded in a high-Q micro-cavity. Two superconducting reservoirs are coupled to the dot via tunneling. The Hamiltonian of the

present system reads

$$H = H_0 + H(t), \quad (1)$$

in which the time-independent H_0 consists of three parts: H_S describes the left and right ordinary BCS superconductors with the energy gap function $\Delta_{L,R}$ and band width W

$$H_S = \sum_{k\sigma\alpha=L,R} \epsilon_k c_{k\sigma\alpha}^\dagger c_{k\sigma\alpha} + \sum_{k\alpha=L,R} [\Delta_\alpha c_{k\uparrow\alpha}^\dagger c_{-k\downarrow\alpha}^\dagger + h.c.]. \quad (2)$$

We assume $\Delta_\alpha = \Delta \exp(i\Phi_\alpha)$, *i.e.*, two superconductors have the same energy gap but the different phase. H_D stands for the Hamiltonian of single energy level quantum dot with the on-site Coulomb interaction

$$H_D = \sum_{\sigma} \epsilon_{\sigma} d_{\sigma}^{\dagger} d_{\sigma} + U n_{\uparrow} n_{\downarrow}. \quad (3)$$

Note that the Zeeman splitting due to the external magnetic field B between two spin states is $h \equiv \epsilon_{\uparrow} - \epsilon_{\downarrow} = g\mu_B B$, and U is on-site Coulomb interaction. H_T is the tunneling Hamiltonian between the superconductors and the quantum dot

$$H_T = \sum_{k\sigma\alpha} [T_{k\sigma\alpha} c_{k\sigma\alpha}^\dagger d_{\sigma} + T_{k\sigma\alpha}^* d_{\sigma}^\dagger c_{k\sigma\alpha}]. \quad (4)$$

The most important term $H(t)$ denotes transitions between the different spin states of the dot, which can be induced by a two-photo Raman process[24]. If we treat the strong laser field classically, $H(t)$ has the following form

$$H(t) = r[d_{\uparrow}^\dagger d_{\downarrow} \exp(-i\omega t) + h.c.]. \quad (5)$$

Here r and ω are the classical Rabi frequency and the σ_- polarized laser frequency, respectively. This σ_- polarized laser acts as the two pumping forces with the phase difference $\pi/2$. It must be pointed out that neglecting the on-site Coulomb interaction and replacing the superconductor by normal metal in the above model have been used to generate spin current[18, 19, 24]. Due to the existence of spin-flip term, we have to work in the generalized Nambu \otimes spin space. The standard Keldysh Green's function gives the charge and spin current ($e = \hbar = 1$)

$$I_{c,s}(t) = \int \frac{dE_1}{2\pi} \frac{dE_2}{2\pi} Tr \{ \sigma_{c,s} [G^r(E_1, E_2) \Sigma_L^<(E_2) + G^<(E_1, E_2) \Sigma_L^a(E_2) + h.c.] \} \\ * \exp[-i(E_1 - E_2)t] \quad (6)$$

where $\sigma_c \equiv \frac{1}{2} \begin{pmatrix} \sigma_z & 0 \\ 0 & \sigma_z \end{pmatrix}$, $\sigma_s \equiv \frac{1}{4} \begin{pmatrix} \hat{1} & 0 \\ 0 & -\hat{1} \end{pmatrix}$. $G^r(E_1, E_2)$ and $G^<(E_1, E_2)$ are the Fourier representations of the retarded and lesser Green's functions in the generalized Nambu \otimes spin space with the creation operator $\Psi^\dagger = (d_\uparrow^\dagger, d_\downarrow^\dagger, d_\downarrow^\dagger, d_\uparrow^\dagger)$. They have the usual form $G^r(t_1, t_2) \equiv -i\theta(t_1 - t_2)\langle\{\Psi(t_1), \Psi^\dagger(t_2)\}\rangle$ and $G^<(t_1, t_2) \equiv i\langle\Psi^\dagger(t_2)\Psi(t_1)\rangle$, respectively. $\Sigma_\alpha^{r,<}(E)$ are the retarded and lesser self-energies due to the superconducting lead α . The lesser Green's function is related to the retarded and advanced Green's functions via Keldysh equation $G^< = G^r \Sigma^< G^a$, therefore, the whole problem is reduced to the calculation of the retarded Green's function G^r . This can be done by two steps: we first calculate the time-independent retarded Green's function $G_0^r(E)$ of the quantum dot, then the full Green's function can be obtained exactly from the Dyson equation $G^r(t_1, t_2) = G_0^r(t_1 - t_2) + \int dt G_0^r(t_1 - t) H(t) G^r(t, t_2)$, which has the following Fourier form

$$\begin{pmatrix} G_{2i}^r(E_1, E_2) & G_{1i}^r(E_1, E_2) \end{pmatrix} = \begin{pmatrix} C_{2i} & C_{1i} \end{pmatrix} \hat{A} / \text{Det} \hat{A}, \quad (7)$$

$$\begin{pmatrix} G_{4i}^r(E_1, E_2) & G_{3i}^r(E_1, E_2) \end{pmatrix} = \begin{pmatrix} C_{4i} & C_{3i} \end{pmatrix} \hat{B} / \text{Det} \hat{B}, \quad (8)$$

where the matrices \hat{A} , \hat{B} and the coefficients C_{ij} are defined as

$$\hat{A} = \hat{1} + r^2 \begin{pmatrix} -G_{011}^r(E_1) & G_{012}^r(E_1) \\ G_{021}^r(E_1) & -G_{022}^r(E_1) \end{pmatrix} \begin{pmatrix} G_{033}^r(E_1 - \omega) & G_{034}^r(E_1 - \omega) \\ G_{043}^r(E_1 - \omega) & G_{044}^r(E_1 - \omega) \end{pmatrix}, \quad (9)$$

$$\hat{B} = \hat{1} + r^2 \begin{pmatrix} -G_{033}^r(E_1) & G_{034}^r(E_1) \\ G_{043}^r(E_1) & -G_{044}^r(E_1) \end{pmatrix} \begin{pmatrix} G_{011}^r(E_1 + \omega) & G_{012}^r(E_1 + \omega) \\ G_{021}^r(E_1 + \omega) & G_{022}^r(E_1 + \omega) \end{pmatrix}, \quad (10)$$

$$C_{ij} = \begin{cases} 2\pi G_{0ij}^r(E_1) \delta(E_1 - E_2) \\ \text{if } i, j = 1, 2 \text{ or } i, j = 3, 4 \\ 2\pi r [G_{0i1}^r(E_1) G_{03j}^r(E_1 - \omega) - G_{0i2}^r(E_1) G_{04j}^r(E_1 - \omega)] \delta(E_1 - E_2 - \omega) \\ \text{if } i = 1, 2 \text{ and } j = 3, 4 \\ 2\pi r [G_{0i3}^r(E_1) G_{01j}^r(E_1 + \omega) - G_{0i4}^r(E_1) G_{02j}^r(E_1 + \omega)] \delta(E_1 - E_2 + \omega) \\ \text{if } i = 3, 4 \text{ and } j = 1, 2 \end{cases}. \quad (11)$$

Substituting the retarded Green's function into Eq.(6), we can obtain the charge and spin current formulae directly, which have the following form

$$I_c = \int \frac{dE}{2\pi} \text{Tr} \{ \sigma_c [\hat{F}(E) f + \hat{F}^+(E) (f^+ - f) + \hat{F}^-(E) (f^- - f) + h.c.] \}, \quad (12)$$

$$I_s = \int \frac{dE}{2\pi} \text{Tr} \{ \sigma_s [\hat{F}^+(E)(f^+ - f) + \hat{F}^-(E)(f^- - f) + h.c.] \}. \quad (13)$$

Here $\hat{F}(E) = G^r(E, E)[\Sigma_L^a(E) - \Sigma_L^r(E)] + \int \frac{dX}{2\pi} \{ G^r(E, X)[\Sigma^a(X) - \Sigma^r(X)]G^a(X, E)\Sigma_L^a(E) \}$,
 $\hat{F}^+(E) = \int \frac{dX}{2\pi} \{ G^r(E, X)\tau_1[\Sigma^a(X) - \Sigma^r(X)]\tau_1 G^a(X, E)\tau_2 \Sigma_L^a(E)\tau_2 \}$, $\hat{F}^-(E) = \int \frac{dX}{2\pi} \{ G^r(E, X)\tau_2[\Sigma^a(X) - \Sigma^r(X)]\tau_2 G^a(X, E)\tau_1 \Sigma_L^a(E)\tau_1 \}$ [25]. The matrices τ_1 and τ_2 are defined as $\tau_1 = \begin{pmatrix} \hat{1} & 0 \\ 0 & 0 \end{pmatrix}$, $\tau_2 = \begin{pmatrix} 0 & 0 \\ 0 & \hat{1} \end{pmatrix}$ respectively. f is the well-known Fermi distribution function $f \equiv 1/(\exp(\beta E) + 1)$ and $f^\pm \equiv f(E \pm \omega)$. Eq.(12) shows that the charge current comes from two parts: the dissipationless supercurrent $I_{sc} = \int \frac{dE}{2\pi} \text{Tr} \{ \sigma_c [\hat{F}(E)f + h.c.] \}$ involving in the transfer of coherent Cooper pairs and dissipative quasi-particle current $I_{qc} = \int \frac{dE}{2\pi} \text{Tr} \{ \sigma_c [\hat{F}^+(E)(f^+ - f) + \hat{F}^-(E)(f^- - f) + h.c.] \}$. Since the Cooper pair is singlet, only can the spin current be carried by the dissipative quasi-particle process. We must stress that the above charge and spin current formulae go beyond the adiabatic approximation, and are valid for the arbitrary temperature and frequency and strength of the laser field since we can calculate the retarded Green's function exactly.

We first consider the non-interacting case on the quantum dot. The time-independent retarded Green's function for the quantum dot is given by

$$G_0^r(E) = \{ E\hat{1} - H_D - \sum_{\alpha} \Sigma_{\alpha}^r(E) \}^{-1}, \quad (14)$$

where $H_D \equiv \begin{pmatrix} \epsilon_{\uparrow} & 0 & 0 & 0 \\ 0 & -\epsilon_{\downarrow} & 0 & 0 \\ 0 & 0 & \epsilon_{\downarrow} & 0 \\ 0 & 0 & 0 & -\epsilon_{\uparrow} \end{pmatrix}$, $\Sigma_{\alpha}^r(E) \equiv -\frac{i\Gamma_{\alpha}\xi_E}{2\sqrt{E^2 - \Delta^2}} \begin{pmatrix} E & -\Delta_{\alpha} & 0 & 0 \\ -\Delta_{\alpha}^* & E & 0 & 0 \\ 0 & 0 & E & \Delta_{\alpha} \\ 0 & 0 & \Delta_{\alpha}^* & E \end{pmatrix}$ with $\xi_E = 1$

for $E > -\Delta$ and $\xi_E = -1$ otherwise. Γ_{α} is the linewidth function which is assumed to be a constant under wide-band limit. Once having the above time-independent retarded Green's function, we can obtain the full Green's function and then charge and spin current by using Eqs.(7-13). Fig.1(a) shows the Josephson current I_{sc} versus the two superconducting phase difference $\Phi \equiv \Phi_R - \Phi_L$ of two superconductors for the various polarized laser frequencies ω . Our system is in the π -state when the laser frequency is lower. As the frequency ω increases, the π state will turn into a 0-state first, *i.e.*, a dynamic $\pi - 0$ transition happens. For a higher frequency the 0-state will experience a dynamic $0 - \pi$ transition and become a π -state again. This result can be seen clearly in Fig.2, which exhibits the Josephson current I_{sc} as

a function of frequency ω . In Fig.1(b) we plot the Josephson current I_{sc} versus the phase difference Φ of two superconductors for the different polarized laser strength r . The similar $0-\pi$ transition can be modulated by the strength r of laser field. Fig.3 describes the pumped quasi-particle charge current I_{qc} versus the laser frequency. We find that this pumped quasi-particle charge current is very small for the lower frequency and increases with frequency non-monotonously. The pumped quasi-particle charge current can be understood from the co-tunneling process. For example, a spin down electron 1 in the left superconductor tunnels into the quantum dot, then a Cooper pair splits into two electrons 2 and 2' with the opposite spin. The spin down electron 2' fills in the original position of the electron 1, while spin up electron 2 will tunnel into the quantum dot. Finally, the spin down electron 1 and spin up electron 2 in the quantum dot change their spin by absorption and emitting a photon, respectively, and form a pair to enter the right superconductor. This co-tunneling process is depicted in the inset of Fig.3. Our numerical calculation shows that the pumped quasi-particle charge current always satisfies $I_{qc11} = I_{qc22} = I_{qc33} = I_{qc44}$ and gives no spin current if only the laser frequency is lower than 2Δ . However, the pumped spin current appears once the laser frequency is higher than 2Δ . The result can be seen from Fig.4. Obviously, this spin current arises from the additional spin-flip process which is plotted in the inset of Fig.4: A spin-down electron from both superconducting leads can tunnel into the quantum dot, and due to the σ_- polarized laser field it absorbs a photon and becomes a spin-up electron. If the laser frequency is higher than 2Δ , this spin-up electron can tunnel out of the scattering region and goes into the superconducting leads as a quasi-particle. In the Fig.1(c) we plot the spin current I_s versus the two superconducting phase difference Φ . In contrast to the Josephson supercurrent, the pumped spin current I_s is an even function of Φ .

Finally, we briefly consider there exists a strong Coulomb interacting case ($U \rightarrow \infty$) in the quantum dot. We focus on the strong coupling limit $\Delta \ll T_K$, where the Kondo effect becomes important and leads to a positive pairing correlation on the quantum dot. This situation is well described in the slave-boson language. The physical electron operator d_σ can be replaced by $b^+ f_\sigma$, where b and f_σ being the standard boson and fermion annihilation operators standing for the empty ($n_\uparrow = 0, n_\downarrow = 0$) and singly occupied ($n_\uparrow = 1, n_\downarrow = 0$) or ($n_\uparrow = 0, n_\downarrow = 1$). We then introduce the constriction which prevents double occupancy in the quantum dot by means of Lagrange multiplier λ . The resulting model can be solved

within the mean-field approach. Two constants b and λ can be calculated from the following self-consistent equations

$$-i \int \frac{dE}{2\pi} \text{Tr} G_0^<(E) + b^2 = 1, \quad (15)$$

$$\lambda b^2 = i \int \frac{dE}{2\pi} \sum_{\alpha} \text{Tr} \{ G_0^r(E) \Sigma_{\alpha}^<(E) + G_0^<(E) \Sigma_{\alpha}^a(E) \}. \quad (16)$$

Having constants b , λ and using Eqs.(7-13), we can obtain the full Green's function and finally give the charge and spin current in the strong coupling limit. Fig.5 shows the Josephson current versus the two superconducting phase difference Φ of two superconductors for the various polarized laser frequencies ω . It is found that as the frequency ω increases, the system will go from a π -state to a 0-state. This means that the strong Coulomb interaction will not affect the dynamic $0 - \pi$ transition.

In summary, we have presented a charge and spin pumping theory of quantum dot coupled to two superconducting leads. The σ_- polarized laser field serving as the pumping forces can induce the dynamic $0 - \pi$ transition and spin current. In contrast to the Josephson supercurrent, the pumped spin current is an even function of the phase difference between two superconductors. The strong Coulomb interaction in the quantum dot does not change the dynamic $0 - \pi$ transition behavior. Since the system under investigation is within the reach of the present nano-technology, we hope that the present theory can stimulate the further study of cavity superconducting electronics and spintronics.

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- [25] The integral of the variable X can be performed easily since there exists δ function in the retarded Green's function.

Figure Captions

Fig.1. (a) Josephson current-phase relation for various laser frequencies ω . $\omega = 0, 0.5, 1.5, 2.5$ correspond to solid, dash, dash-dot, and short-dash lines, $\epsilon_{\uparrow} = \epsilon_{\downarrow} = 0$, $r = 0.6$,

$\Gamma_L = \Gamma_R = 0.1$. (b) Josephson current-phase relation for various laser strength r . $r = 0.1, 0.3, 0.5$ correspond to solid, dash, dot lines, $\epsilon_\uparrow = -0.2$, $\epsilon_\downarrow = -0.4$, $\omega = 0.3$, $\Gamma_L = \Gamma_R = 0.1$. (c) The pumped spin current versus the phase difference Φ between two superconductors. We have set $\omega = 2.5$ and other parameters are the same as those in Fig.1(a). Note that all parameters are in unit of superconducting energy gap Δ .

Fig.2. Josephson current I_{sc} versus frequency ω . Here $\Phi = \pi/2$ and other parameters are the same as in Fig.1(a).

Fig.3. Pumped quasi-particle charge current I_{qc} as a function of ω . Here $\epsilon_\uparrow = 0.9$, $\epsilon_\downarrow = 0.5$, $r = 0.6$ and other parameters are the same as in Fig.2(a). Inserted: co-tunneling process for the pumped quasi-particle charge current.

Fig.4. Pumped quasi-particle spin current I_s as a function of ω . Parameters are the same as in Fig.3. Inserted: spin-flip process for the pumped spin current.

Fig.5. Josephson current-phase relation for various laser frequencies ω with a strong Coulomb interaction in the quantum dot. $\omega = 0.6, 1.2, 2.2, 2.8$ correspond to solid, dash, dot, dash-dot line. Other parameters are $\epsilon = -2$, $\Gamma_L = \Gamma_R = 0.5$, $r = 2$, $\Phi \equiv \pi/2$, $\Delta = 0.002$, $W = 100$, $\beta = 500$.

Fig. 1

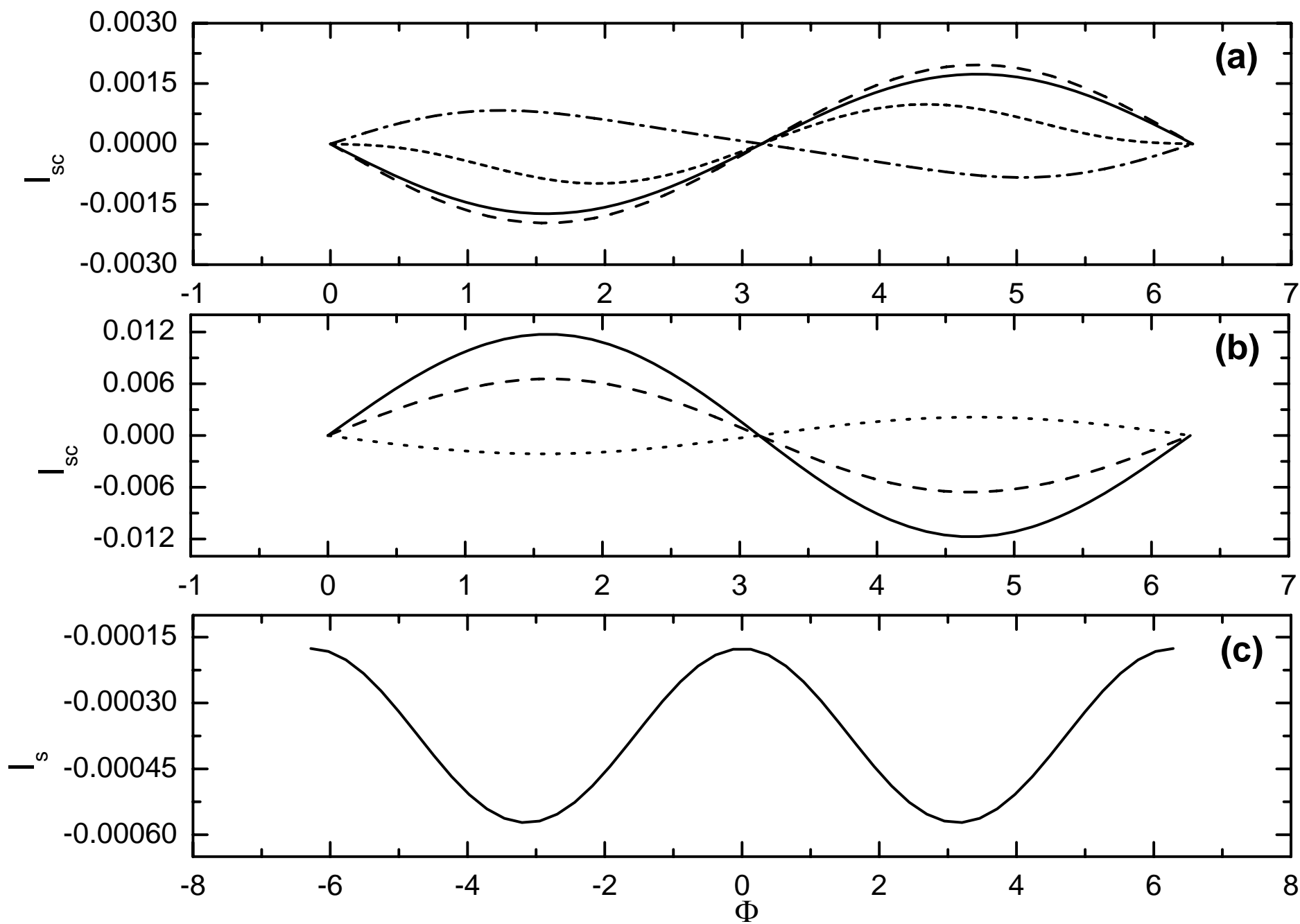


Fig. 2

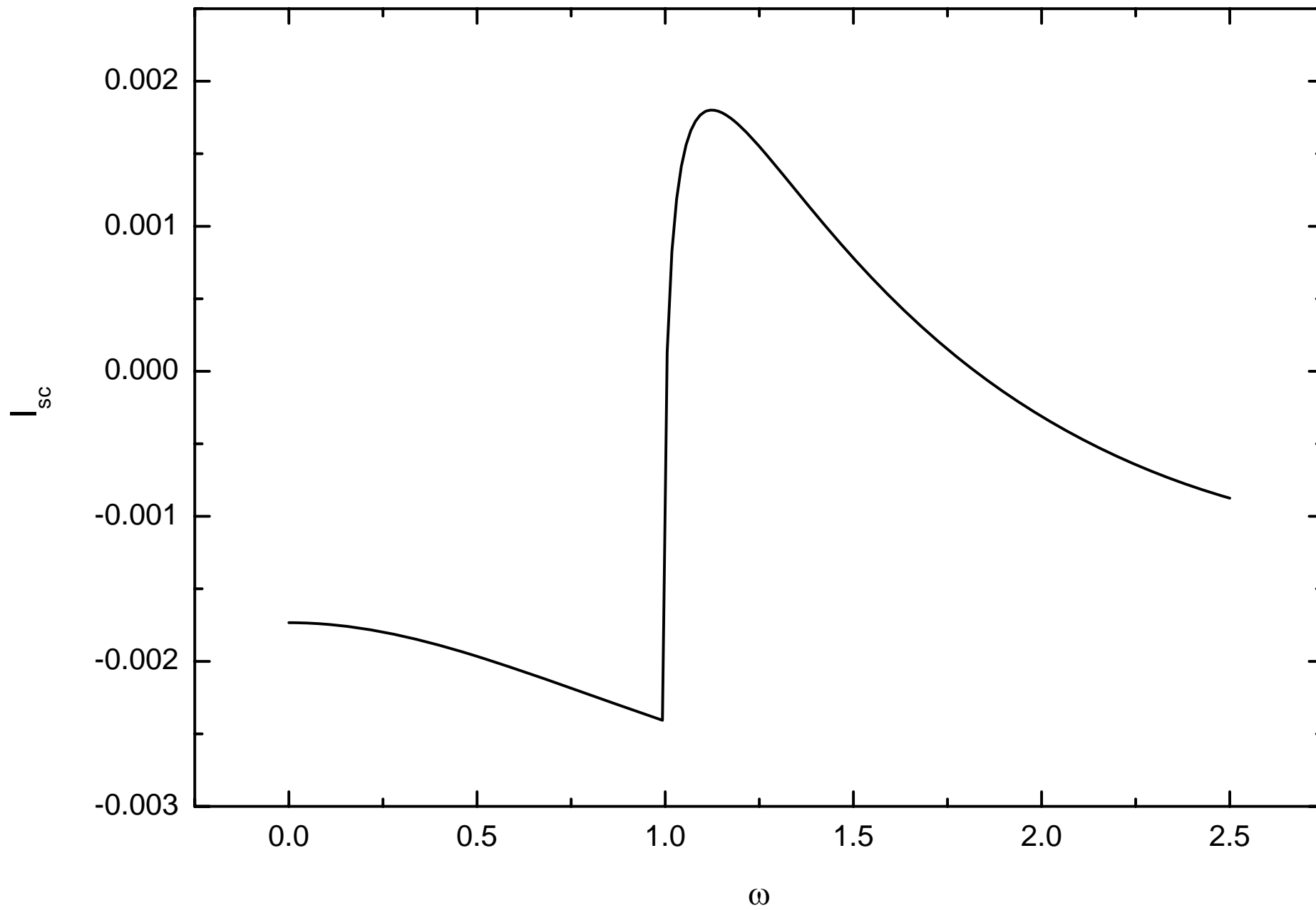


Fig. 3

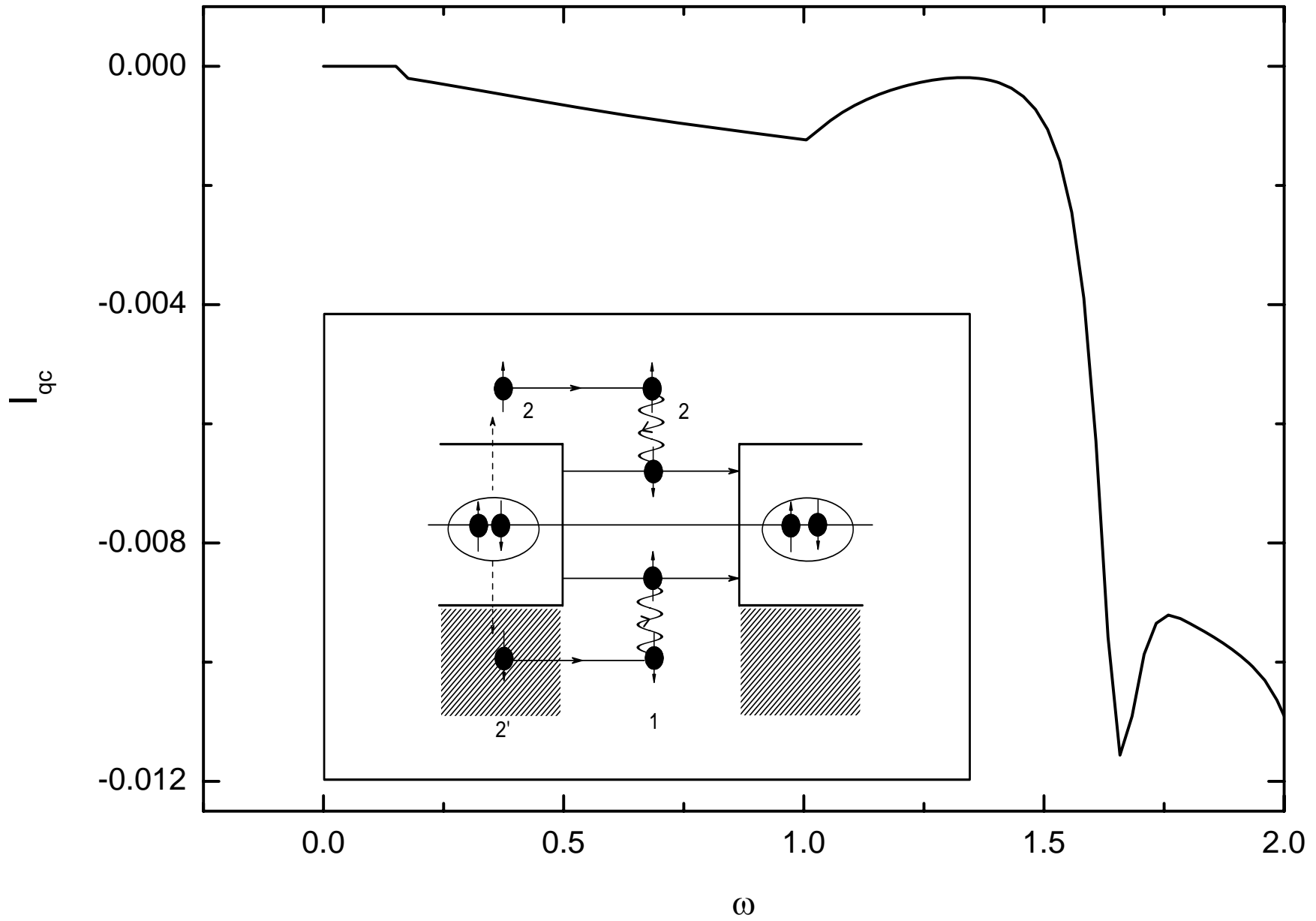


Fig. 4

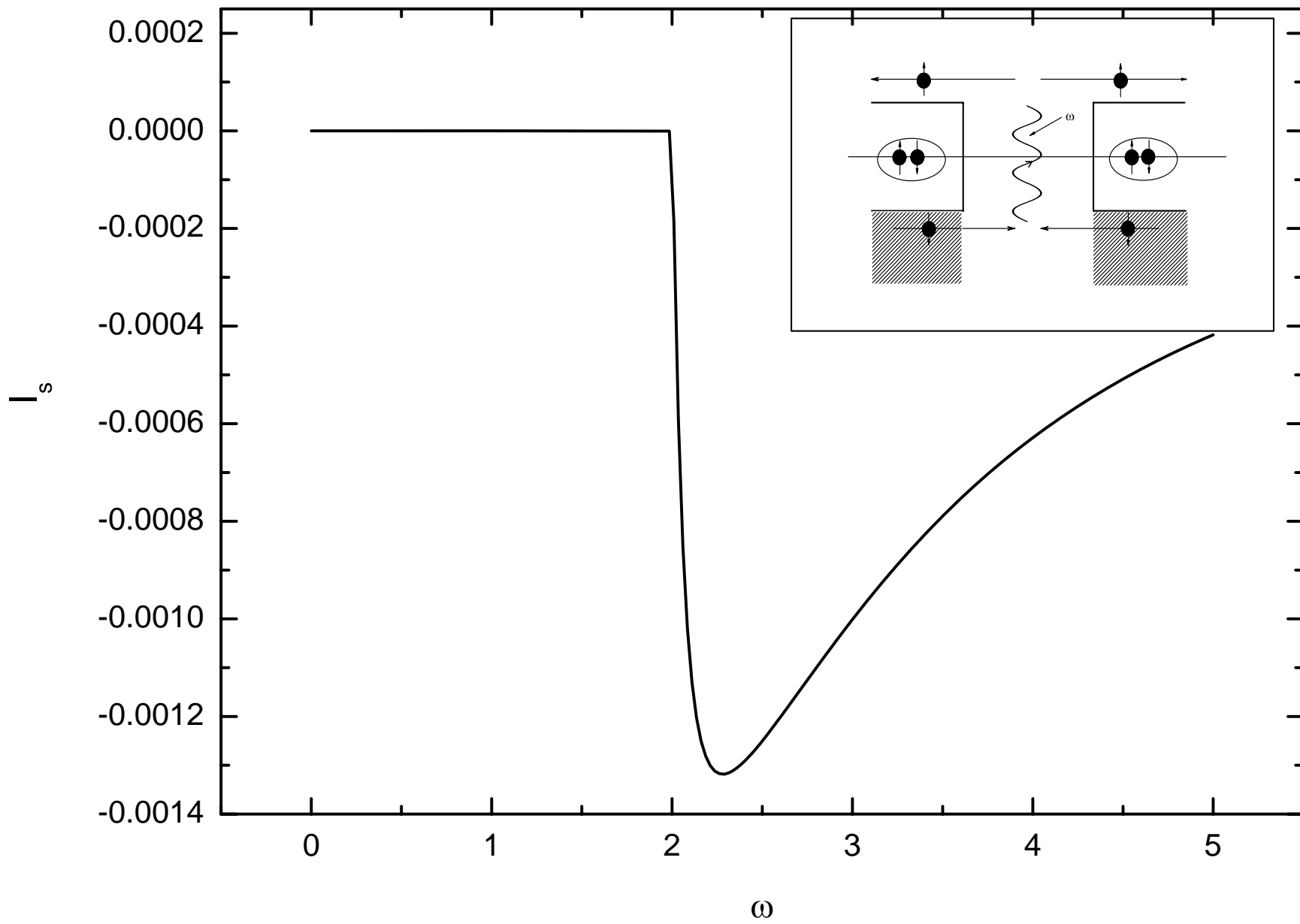


Fig. 5

